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ELECTRICAL & COMPUTER ENGINEERING
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A Robust Algorithm for Online Switched System Identification

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Background

- Application of system identification
  - Adaptive control
  - Fault detection
  - Motion/texture video segmentation
  - Applications involving autoregressive model

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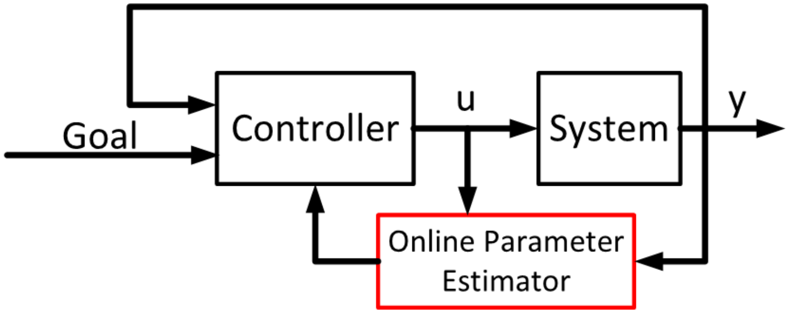
Adaptive Control



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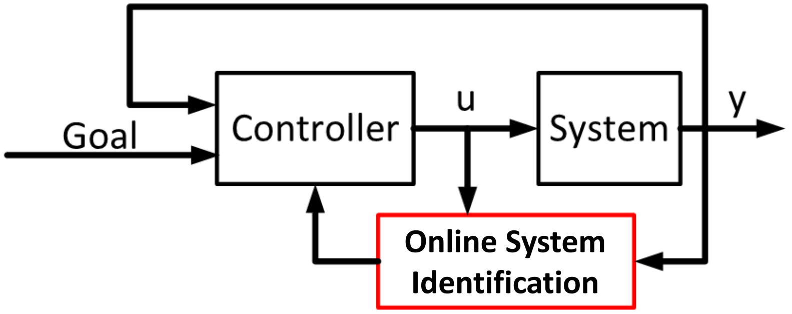

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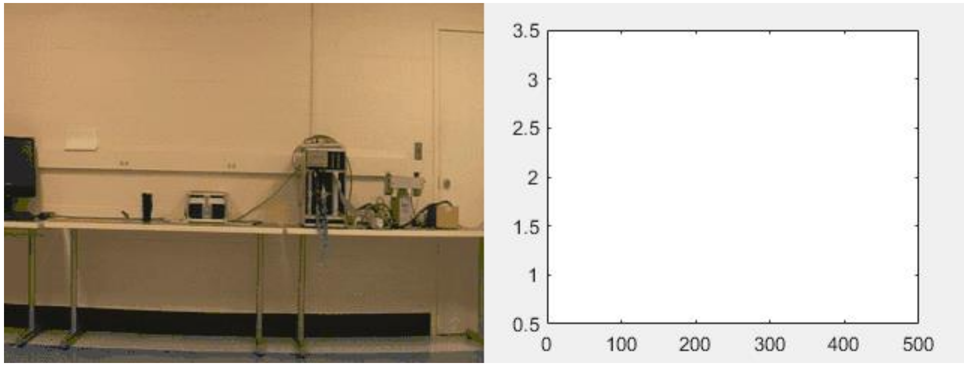

Ozay et al. (2015, 2012)

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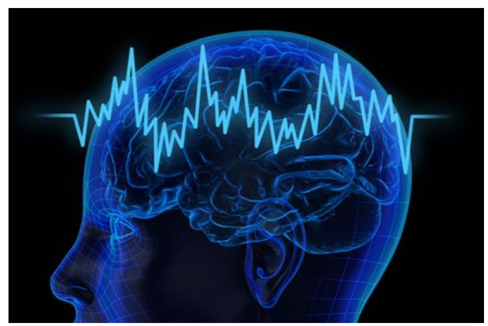
earthquake prediction
Dai et al. (2015)



brain electrical activity mapping
Ogawa et al. (1993)

financial time series analysis
Aas and Dimakos (2004)

Background

- Status quo of online switched sys. id. (Bako et al. (2011), Goudjil et al.(2016))
  - Sensitive to initialization
  - Lack nice theoretical guarantees

- Our main contributions:
  - Proposed an algorithm that is robust to initialization
  - Proved theoretical guarantees under local initialization

Prelims: SARX System Identification

- Switched AutoRegressive eXogenous (SARX) System

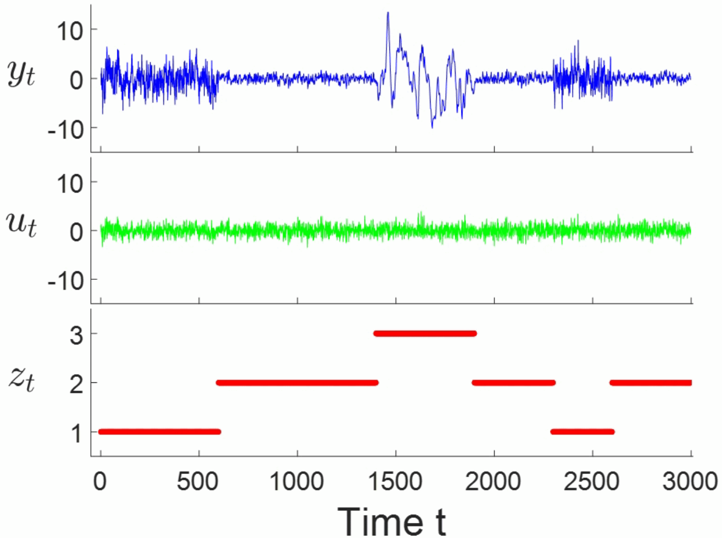
$$y_t = \sum_{j=1}^{o_a} a_j(z_t) y_{t-j} + \sum_{k=1}^{o_c} c_k(z_t) u_{t-k} + n_t$$

  - $z_t \in \{1, \ldots, m\}$ indexes the active subsystem at time t
  - Example: $m = 3$

Subsystem 1: $y_t = 0.2y_{t-1} + 0.24y_{t-2} + 2u_{t-1}$

Subsystem 2: $y_t = 0.7y_{t-1} - 0.12y_{t-2} + 0.5u_{t-1}$

Subsystem 3: $y_t = 1.7y_{t-1} - 0.72y_{t-2} + 0.5u_{t-1}$



Prelims: SARX System Identification

- Switched AutoRegressive eXogenous (SARX) System

$$y_t = \sum_{j=1}^{o_a} a_j(z_t) y_{t-j} + \sum_{k=1}^{o_c} c_k(z_t) u_{t-k} + n_t$$

  - $z_t \in \{1, \ldots, m\}$ indexes the active subsystem at time t
  - **Assume**, $\mathbb{E}[n_t] = 0, |n_t| \leq n_{\max}$, and $n_{\max}, o_a, o_c, m$ are known
  - **Simplification**: let $\mathbf{w}_{z_t} = [a_{1:o_a}(z_t), c_{1:o_c}(z_t)]^\intercal, \boldsymbol{\phi}_t = [y_{t-1:t-o_a}, u_{t-1:t-o_c}]^\intercal$, then

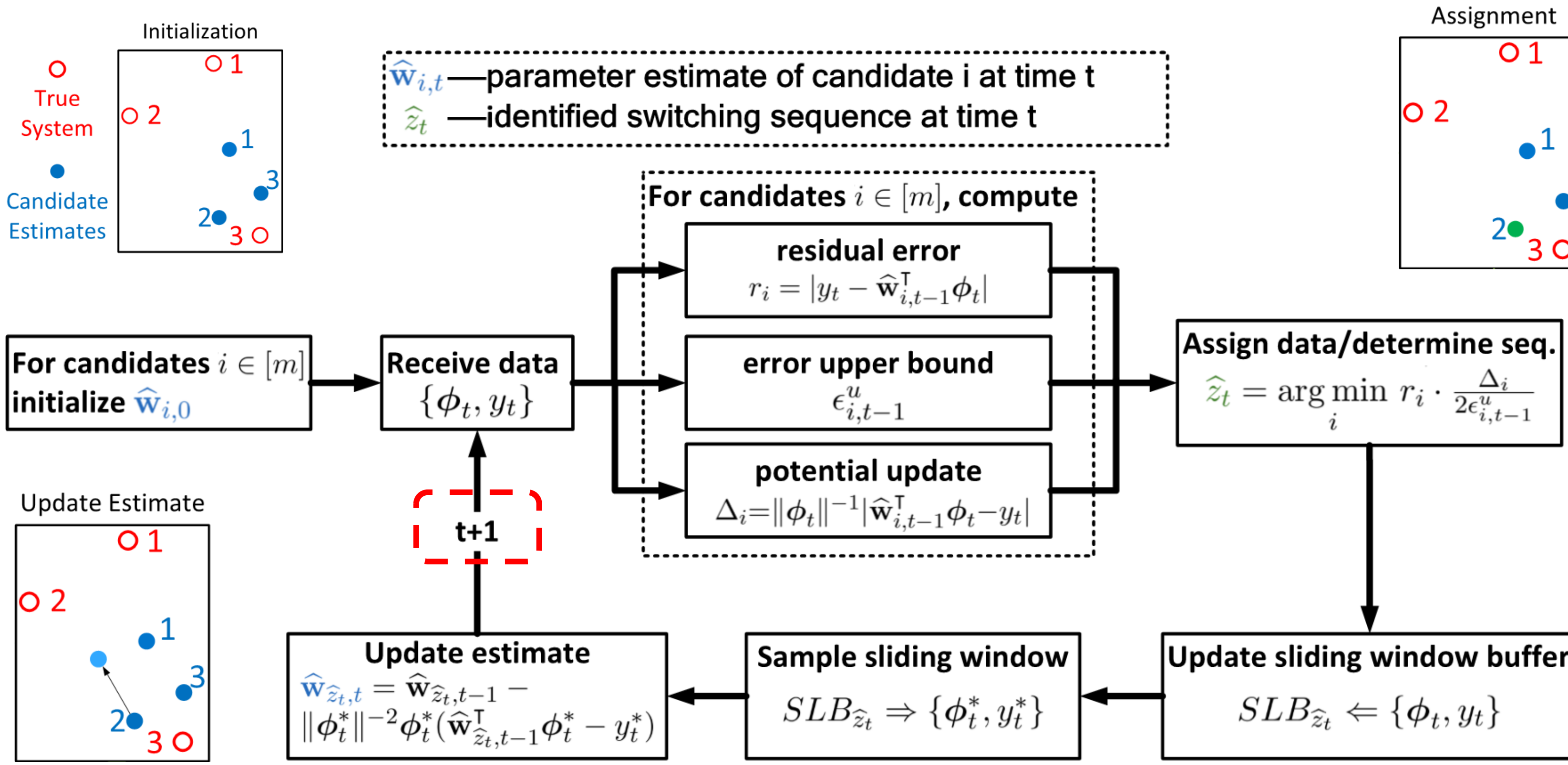
$$y_t = \mathbf{w}_{z_t}^\intercal \boldsymbol{\phi}_t + n_t$$

- Goal of Sys. Id.:

Given the output y_t and input u_t (or $\{\phi_t, y_t\}$, equivalently),

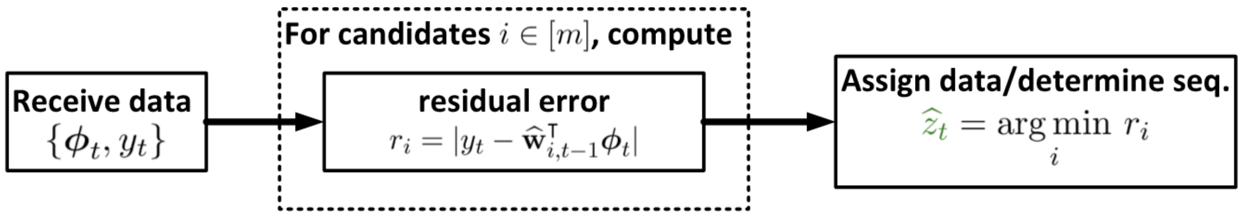
  - Identify switching sequence $\{z_t\}$
  - Estimate parameter a and c for each subsystem, i.e. $\mathbf{w}_{z_t}$
  - Particularly, in a online fashion

Algorithm: Overview

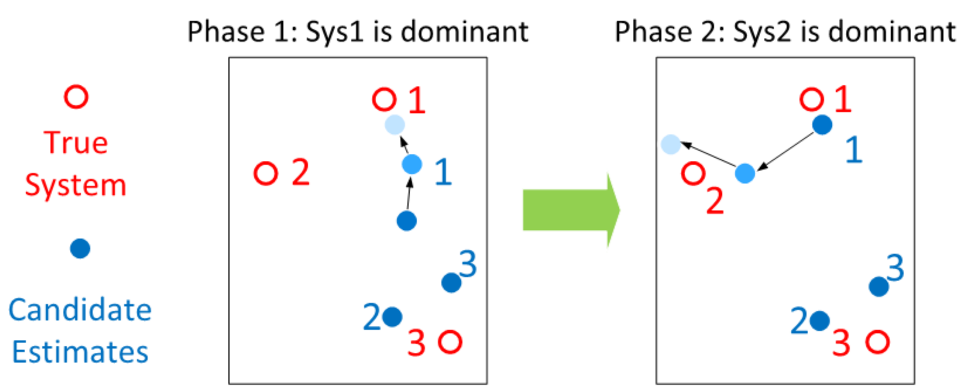


Algorithm: Make Assignment

- What if we assign data based on minimum residual error?

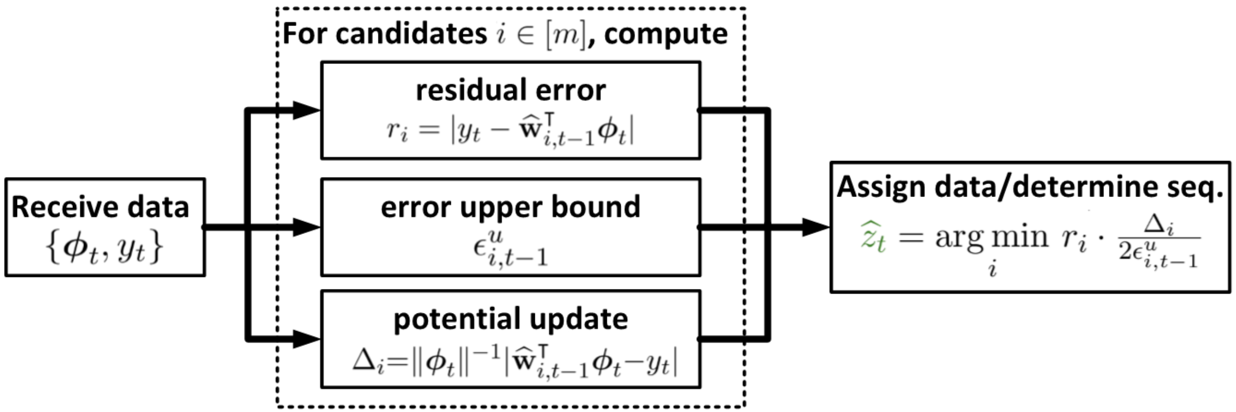


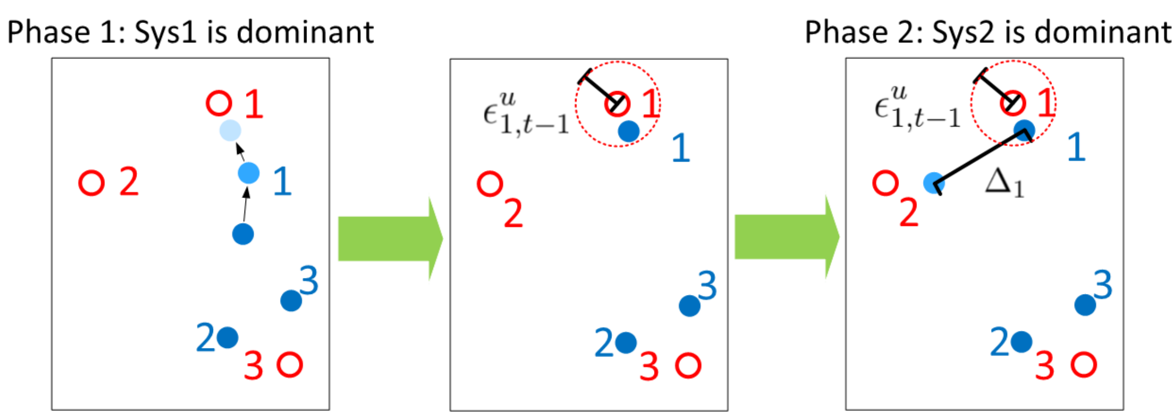
  - Sensitive to initialization: previously well converged candidate may shift to learn a new subsystem!
  - Failure illustration:



Algorithm: Make Assignment

- Our strategy





Theoretical Guarantees

Theorem 1 (Local Convergence). *WLOG, if* $\forall i, \|\boldsymbol{\epsilon}_{i,0}\| \equiv \|\mathbf{w}_i - \widehat{\mathbf{w}}_{i,0}\| \leq \epsilon_0$, *then under certain assumptions, with probability at least* $1 - 2m\sqrt{\frac{N_R}{\epsilon'^2}\left(\epsilon_0^2 + \frac{N_R}{S_{\min}^2}\right)}$, *we have the following results:*

- *We can correctly identify the switching sequence, i.e.* $\forall t, \widehat{z}_t = z_t$
- *We have the following convergence bounds in mean square sense:*

decaying part

$$\mathbb{E}\left[\|\boldsymbol{\epsilon}_{i,t}\|^2\right] \leq \left(1-\kappa_{\max}^{-2}\right)^t c_1 + N_R \frac{\kappa_{\max}^2}{F_{\min}^2}\left[1 - d_1\left(1-\kappa_{\max}^{-2}\right)^t\right]\sigma_n^2$$

$$\mathbb{E}\left[\|\boldsymbol{\epsilon}_{i,t}\|^2\right] \geq \left(1-\xi_{\min}^{-2}\right)^t c_2 + N_R \frac{\xi_{\min}^2}{F_{\max}^2}\left[1 - d_2\left(1-\xi_{\min}^{-2}\right)^t\right]\sigma_n^2$$

If $t \to \infty$, *then*

$$N_R \frac{\xi_{\min}^2}{F_{\max}^2}\sigma_n^2 \leq \mathbb{E}\left[\|\boldsymbol{\epsilon}_{i,t}\|^2\right] \leq N_R \frac{\kappa_{\max}^2}{F_{\min}^2}\sigma_n^2$$

Numerical Simulations

- Evaluation of convergence bounds

$$\mathbb{E}\left[\|\boldsymbol{\epsilon}_{i,t}\|^2\right] \leq \left(1-\kappa_{\max}^{-2}\right)^t c_1 + N_R \frac{\kappa_{\max}^2}{F_{\min}^2}\left[1 - d_1\left(1-\kappa_{\max}^{-2}\right)^t\right]\sigma_n^2$$

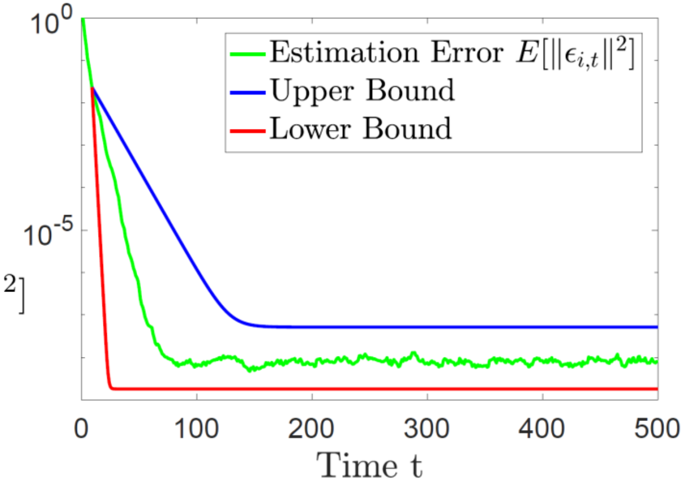
$$\mathbb{E}\left[\|\boldsymbol{\epsilon}_{i,t}\|^2\right] \geq \left(1-\xi_{\min}^{-2}\right)^t c_2 + N_R \frac{\xi_{\min}^2}{F_{\max}^2}\left[1 - d_2\left(1-\xi_{\min}^{-2}\right)^t\right]\sigma_n^2$$

- Consider a single subsystem

$$y_t = 0.7y_{t-1} - 0.12y_{t-2} + u_{t-1} + n_t$$

with $n_t \sim \mathcal{N}(0, \sigma_n^2)$, $\sigma_n = 10^{-4}$, and $u_t \sim \mathcal{N}(0,1)$

- Take the average of $\|\boldsymbol{\epsilon}_{i,t}\|^2$ over 50 runs as $\mathbb{E}[\|\boldsymbol{\epsilon}_{i,t}\|^2]$

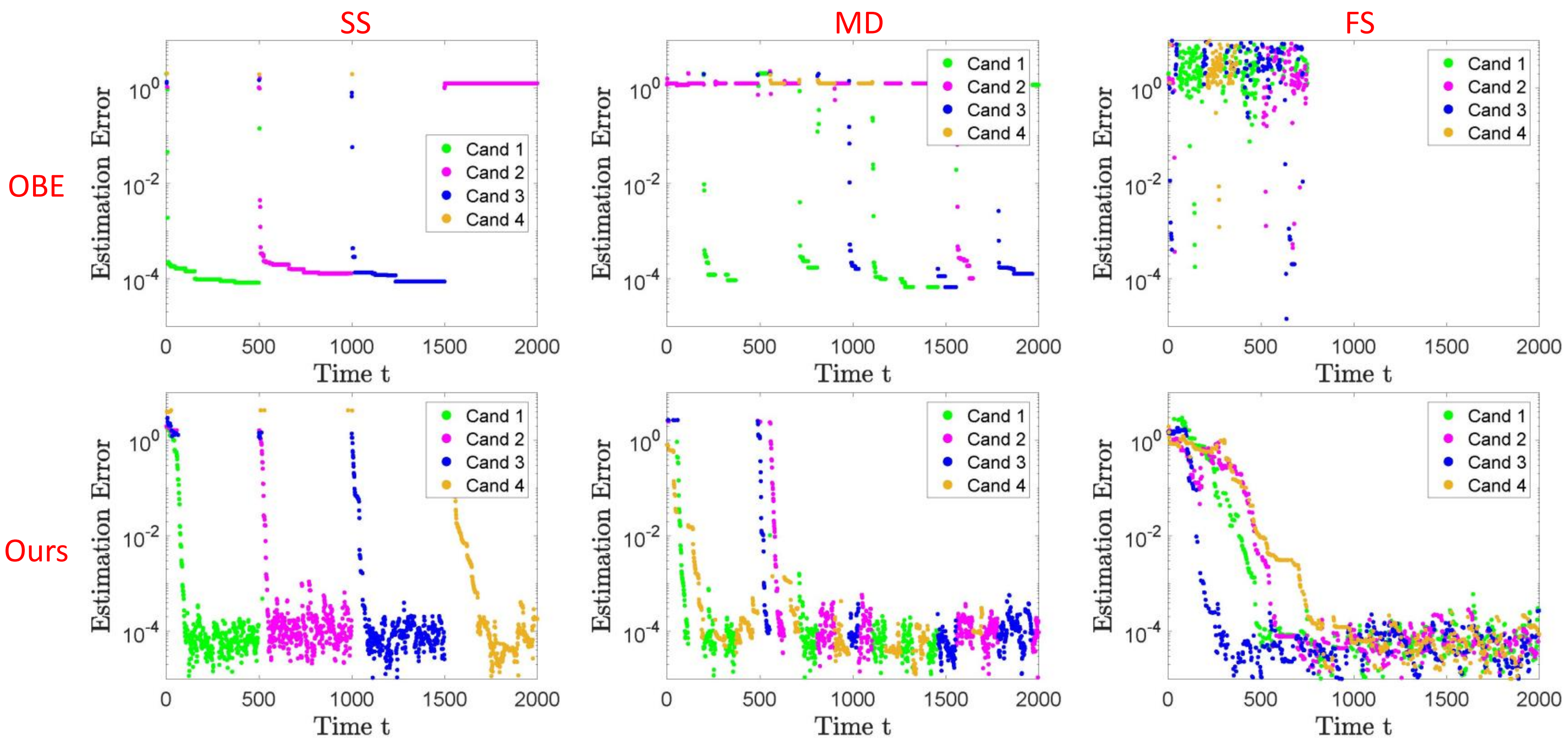


Numerical Simulations

- Performance Evaluation and Comparison (Single realization)
  - Consider SARX system with 4 subsystems
    - Subsystem 1: $y_t = 0.2y_{t-1} + 0.24y_{t-2} + 2u_{t-1} + n_t$
    - Subsystem 2: $y_t = 0.7y_{t-1} - 0.12y_{t-2} + 1u_{t-1} + n_t$
    - Subsystem 3: $y_t = -1.4y_{t-1} - 0.53y_{t-2} + 1u_{t-1} + n_t$
    - Subsystem 4: $y_t = 1.7y_{t-1} - 0.72y_{t-2} + 0.5u_{t-1} + n_t$
  - with $n_t \sim \text{truncate}(\mathcal{N}(0, \sigma_n^2), [-3\sigma_n, 3\sigma_n]), \sigma_n = 10^{-4}$, and $u_t \sim \mathcal{N}(0,1)$
  - Consider 3 switching patterns
    - Slow Switching (SS)
      - $z_t = \lceil t/500 \rceil$
    - Minimum Dwell Time (MD)
      - Each subsystem dominates for $30 + Geom(1/16)$, then switch to a new subsystem equally likely
    - Fast Switching (FS)
      - $\forall i, P(z_t = i) = 0.25, z_t \perp\!\!\!\perp z_s$

Numerical Simulations

- Comparison with the Outer Bounding Ellipsoid (OBE) algorithm Goudjil et al.(2016)



Numerical Simulations

- Performance Evaluation and Comparison (Multiple Realizations)
  - Same four subsystems under all combinations of $\{SS, MD, FS\}$ and $\sigma_n = \{10^{-1}, 10^{-2}, 10^{-3}\}$
  - For each combination, run 100 realizations, and for the $i\text{th}$ realization, randomly generate system parameters and compute

$$\text{FE}(i) = \frac{1}{m}\sum_{j=1}^{m} \|\boldsymbol{\epsilon}_{j,T}\| \qquad \text{CER}(i) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{1}\{z_t \neq \widehat{z}_t\}$$

  - The average FE and CER are given below

	Ours	OBE	Ours	OBE
	FE	FE	CER	CER
SS, 10^{-1}	8.4x10^{-1}	8.7x10^{-1}	56.3%	59.1%
SS, 10^{-2}	2.8x10^{-2}	8.2x10^{-1}	22.1%	55.5%
SS, 10^{-3}	9.0x10^{-3}	8.2x10^{-1}	8.35%	56.4%
MD, 10^{-1}	4.3x10^{-1}	5.2x10^{-1}	47.5%	50.3%
MD, 10^{-2}	4.0x10^{-2}	2.8x10^{-1}	11.3%	31.3%
MD, 10^{-3}	9.4x10^{-3}	2.4x10^{-1}	4.91%	28.8%
FS, 10^{-1}	2.6x10^{-1}	6.8x10^{-1}	39.3%	53.9%
FS, 10^{-2}	6.0x10^{-2}	1.5x10^{-1}	11.7%	22.1%
FS, 10^{-3}	5.8x10^{-2}	1.8x10^{-1}	8.93%	18.9%

Conclusion & Future Plan

- Conclusion
  - Proposed an online SARX system identification algorithm that is robust to initialization using a novel criterion to make assignment
  - Showed the theoretical guarantees: (i) exact switching sequence identification and (ii) convergence bounds on estimation error, with local initialization
  - Simulations demonstrate satisfactory performance under various experiment setups
- Future Work
  - Theory: relax assumptions, global convergence
  - Application: Video segmentation, drone controller identification
  - Extension: Input design

Thank you!